GENERAL APTITUDE

Q. No. 1 - 5 Carry One Mark Each

1. Daytime temperatures in Delhi can ___ 40°C

(A) Peak (B) reach (C) get (D) stand

Key: (B)

2. The growth rate of ABC Motors in 2017 was the same ___ XYZ Motors in 2016.

(A) as those of (B) as that of (C) as that off (D) as off

Key: (B)

3. Suresh wanted to lay a new carpet in his new mansion with an area of 70 x 55 sq. mts. However an area of 550 sq. mts had to be left out for flower pots. If the cost of carpet is Rs. 50 per sq. mts, how much money (in Rs.) will be spent by Suresh for the carpet now?

(A) 1,65,000 (B) Rs. 1,92,500 (C) Rs. 1,27,500 (D) Rs. 2,75,000

Key: (A)

Sol: Area of carpet $=(70\times55)$sq.mts. = 3850 sq. mts.

Given, left out area for flower pots = 550 sq.mts.

$\therefore$ Required area of carpet $=3850-550=3300$ sq.mts.

$\therefore$ Cost of required carpet area $=3300\times50=$ Rs. 1,65,000.

4. A retaining wall with measurements 30 m × 12 m × 6m was constructed with bricks of dimensions 8 cm × 6cm× 6cm. If 60% of the wall consists of bricks used for the construction, the number of bricks is ______ lakhs

(A) 45 (B) 30 (C) 40 (D) 75

Key: (A)

Sol: From the given data, we have

Volume of wall = $=30\times12\times6=2160\text{m}^3$

$=2160\times100\times100\times100\text{cm}^3 \quad [\because 1\text{m}=100\text{cm}]$

$=2160\times10^6\text{cm}^3$

$\therefore$ Volume of one brick $=(8\times6\times6)\text{cm}^3=288\text{cm}^3$

$\therefore$ 60% of volume of wall $= \frac{60}{100} \times 2160 \times 10^6$

$= 216 \times 6 \times 10^6 = 1296 \times 10^6$

Number of bricks used in 60% of volume of wall $= \frac{1296 \times 10^6}{288} = 4.5 \times 10^6 = 45$ lakhs.

5. Hima Das was ___ only Indian athlete to win___ gold for India.

(A) the, a (B) an, a (C) an, the (D) the, an

Key: (A)

Q. No. 6 - 10 Carry Two Marks Each

6. Population of state X increased by x% and the population of state Y increased by y% from 2001 to 2011. Assume that x is greater than y. Let P be the ratio of the population of state X to state Y in a given year. The percentage increase in P from 2001 to 2011 is______.

(A) $x - y$ (B) $\frac{100(x-y)}{100+x}$ (C) $\frac{100(x-y)}{100+y}$ (D) $\frac{x}{y}$

Key: (B)

Sol: Given that,

P be the ratio of the population of state X to state Y in 2001.

Let, the population of state x in 2001= A

Population of state Y in 2001 =B

From (1), $\frac{A}{B} = P \rightarrow (2)$

Now population of state X increased by x% and population of state Y increased by y% from 2001 to 2011.

So from;

$$\frac{A \times \left[\frac{10+x}{100}\right]}{B \times \left[\frac{100+y}{100}\right]} = P^* \rightarrow (3); \text{ where } P^* \rightarrow \text{new ratio.}$$

The pecent increase in P from 2001 to 2011 is

$$\frac{P^{*}-P}{P}\times 100; \text{since } x>y.$$

$$\Rightarrow \text{req percentage increase} = \frac{\left[\dfrac{\dfrac{A[100+x]}{100}}{\dfrac{B[100+y]}{100}}-\dfrac{A}{B}\right]}{\dfrac{A}{B}}$$

$$=\left[\frac{100+x}{100+y}-1\right]\times 100$$

$$=\left[\frac{100+x-100-y}{100+y}\right]\times 100=\frac{100(x-y)}{100+y}$$

7. The Newspaper reports that over 500 hectares of tribal land spread across 28 tribal seetlements in Mohinitampuram forest division have already been “alienated’. A top forest official said, “First the tribals are duped out of their land holdings. Second, the families thus rendered landless are often forced to encroach further into the forests”. On the basis of the information available in the paragraph, ___ is/are responsible for duping the tribals.

(A) The newspaper
(B) Landless families
(C) forest officials
(D) it cannot be inferred who

Key: (D)

8. An oil tank can be filled by pipe X in 5 hours and pipe Y in 4 hours, each pump working on its own. When the oil tank is full and the drainage hole is open, the oil is drained in 20 hours. If initially the tank was empty and someone started the two pumps together but left the drainage hole open, how many hours will it tak for the tank to be filled? (Assume that the rate of drainage is independent of the head)

(A) 2.50
(B) 1.50
(C) 2.00
(D) 4.00

Key: (A)

Sol: From the given data;

Pipe ‘X’ can fill the tank in 5 hours

$$\Rightarrow \text{Pipe 'X'} \rightarrow 1 \text{ hour work} = \frac{1}{5}$$

Pipe ‘Y’ can fill the tank in 4 hours

$$\Rightarrow \text{Pipe 'Y'} \rightarrow 1 \text{hour work} = \frac{1}{4}$$

Let pipe ‘Z’ can emptying the tank in 20 hours.

i.e, $\text{PipeZ} \rightarrow 1\,\text{hour work} = \frac{-1}{20}$

$[X+Y+Z] \rightarrow 1\text{hr work} = \frac{1}{5}+\frac{1}{4}-\frac{1}{20}=\frac{8}{20}$

$\therefore$ Required time $= \frac{1 \rightarrow \text{Total work}}{8/20 \rightarrow 1\,\text{hour work}}$

$= \frac{20}{8} = \frac{5}{2} = 2.5\,\text{hours.}$

9. Mohan, the manager, wants his four workers to work in pairs. No pair should work for more than 5 hours. Ram and John have worked together for 5 hours. Krishna and Amir have worked as a team for 2 hours. Krishna doesnot want to work with Ram. Whom should Mohan allot to work with John, if he wants all the workers to continue working?

(A) Amir
(B) Krishna
(C) Ram
(D) None of three

Key: (B)

Sol: Mohan $\rightarrow$ Manger

Ram John Krishan Amir $\rightarrow$ workers.

No pair should work for more than 5 hours.

Krishna & Amir worked together for 2 hours.

Ram & John have worked together for 5 hours.

Now we have to consider new pairs.

New pair

$$\left.\begin{array}{ccc} \text{Ram} & \leftrightarrow & \text{John} \\ \updownarrow & & \updownarrow \\ \text{Amir} & & \text{Krishna} \end{array}\right\}^{\text{[present pair]}} \rightarrow \text{New pair.; Since Krishna does not want work with ram}$$

After ‘5’ hrs; so

$\text{John}(-)\underset{\substack{\downarrow \\ \text{One pair.}}}{\text{Krishna}}$ want to work with ram.

Ram $\leftrightarrow$ Amir $\rightarrow$ Another pair.

$\therefore$ Mohan Should allot Krishna to work with John.

10. “Popular Hindi fiction, despite – or perhaps because of – its wide reach, often does not appear in our cinema. As ideals that viewers are meant to look up to rather than identify with, Hindi film protagonists usually read books of aspirational value: textbooks, English books, or high value literature.”

Which one of the following CANNOT be inferred from the paragraph above?

(A) Textbooks, English books or high literature have aspirational value, but not popular Hindi fiction

(B) Protagonists in Hindi movies, being ideals for viewers, read only books of aspirational value

(C) People do not look up to writers of textbooks, English books or high value literature

(D) Though popular Hindi fiction has wide reach, it often does not appear in the movies

Key: (B)

CIVIL ENGINEERING

Q. No. 1 to 25 Carry One Mark Each

1. What is the curl of the vector field $2x^2yi + 5z^1j = 4yzk$?

(A) $-14zi - 2x^2k$ (B) $6zi + 4xj - 2x^2k$

(C) $6zi + 8xy + 2x^2yk$ (D) $-14zi + 6yj + 2x^2k$

Key: (A)

Sol: Consider the vector field

$$\vec{F} = 2x^2yi + 5z^2j - 4yzk$$

$$\text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2x^2y & 5z^2 & -4yz \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(-4yz) - \frac{\partial}{\partial z}(5z^2)\right] - j\left[\frac{\partial}{\partial x}(-4yz) - \frac{\partial}{\partial z}(2x^2y)\right] + k\left[\frac{\partial}{\partial x}(5z^2) - \frac{\partial}{\partial y}(2x^2y)\right]$$

$$\Rightarrow \text{Curl } \vec{F} = i[-4z - 10z] - j[-0] + k\left[-2x^2\right]$$

$$\Rightarrow \text{Curl } \vec{F} = -14z\, i - 2x^2k$$

2. Analysis of a water sample revealed that the sample contains the following species.

$CO_3^{2-}, Na^+, H^+, PO_4^3, Al^{3+}, H_2CO_3, CI^-, Ca^{2+}, Mg^{2+}, HCO_3^-, Fe^{2+}, OH^-$

Concentrations of which of the species will be required to compute alkalinity?

(A) $CO_3^{2-}, H^+, HCO_3^-, OH^-$ (B) $H^+, H_2CO_3, HCO_3^-, OH^-$

(C) $CO_3^{2-}, H_2CO_3, HCO_3^-, OH^-$ (D) $CO_3^{2-}, H^+, H_2CO_3, HCO_3^-$

Key: (A)

Sol:
- Alkalinity in water caused by mainly OH^-, CO_3^{2-} and HCO_3^-
- HCO_3^- and H_2CO_3 never come together.

3. The value of the function f(x) is given at n distinct values of x and its value is to be interpolated at the point x* using all the n points.

   The estimate is obtained first by the Lagrange polynomial, denoted by I_L, and then by the Newton polynomial, denoted by I_N.

   Which one of the following statements is correct?

   (A) I_L is always greater than I_N

   (B) No definite relation exists between I_L and I_N.

   (C) I_L is always less than I_N

   (D) I_L and I_N are always equal

Key: (D)

Sol: I_L and I_N are always equal, since the n^{th} degree polynomial generated by the newton's divided difference formula is the exact same polynomial generated by Lagrange formula. Thus, the error is the same.

4. If the fineness modulus of a sample of the fine aggregates is 4.3, the mean size of the particles in the sample is between

   (A) 150 μm and 300 μm (B) 2.36 mm and 4.75 mm

   (C) 300 μm and 600 μm (D) 1.18 mm and 2.36 mm

Key: (D)

Sol:

150μm	-	1
300μm	-	2
600μm	-	3
1.18mm	-	4
2.36mm	-	5
4.78 mm	-	6

5. The command area of a canal grows only one crop, i.e., wheat. The base period of wheat is 120 days and its total water requirement, Δ, is 40 cm. If the canal discharge is 2 m^3/s, the area, in hectares, rounded off to the nearest integer, which could be irrigated (neglecting all losses), is _______.

Key: (5184)

Sol: Base period (B) = 120 days

Delta (Δ) = 40cm=0.40m

Discharge (Q) = 2m^3/sec

$$\text{Duty (D)} = \frac{8.64B}{\Delta} = \frac{8.64 \times 120}{0.40} = 2592$$

$$\text{Discharge (Q)} = \frac{\text{Area}}{\text{Duty}} \Rightarrow A = Q \times D = 2 \times 2592 = 5184 \text{ hectres}$$

6. The characteristic compressive strength of concrete required is a project is 25 MPa and standard deviation in the observed compressive strength expected at site is 4 MPa. The average compressive strength of cubes tested at different water-cement (w/c) ratios using the same material as is used for the project is given in the table.

w/c(%)	45	50	55	60
Average compressive Strength of cubes (MPa)	35	25	20	15

The water-cement ratio (in percent, round off to the lower integer) to be used in the mix is

Key: (46)

Sol: $f_{ck} = 25\text{MPa}$

$f_m = f_{ck} + 1.65\sigma = 25 + 1.65 \times 4 = 31.6\text{MPa}$

From table

f_m	w / c
35	45
31.6	?
25	50

From linear interpolation

$$\frac{35-25}{35-31.6} = \frac{45-50}{45-x}$$

$$45 - x = (45-50) \times \left(\frac{35-31.6}{10}\right)$$

$$45 - x = -1.7 \Rightarrow x = 45 + 1.7 = 46.7$$

7. A solid sphere of radius, r, and made of material with density ρ_s, is moving through the atmosphere (constant pressure, p) with a velocity, v. The net force ONLY due to atmospheric pressure (F_p) acting on the sphere at any time, t, is

(A) $\frac{4}{3}\pi r^3 \rho_s \frac{dv}{dt}$ (B) zero (C) $\pi r^2 p$ (D) $4\pi^2 p$

Key: (B)

8. An earthen dam of height H is made of cohesive soil whose cohesion and unit weight are c and γ, respectively. If the factor of safety against cohesion is F_c, the Taylor's stability number (S_n)

(A) $\frac{\gamma H}{cF_c}$ (B) $\frac{F_c\gamma H}{c}$ (C) $\frac{c}{F_c\gamma H}$ (D) $\frac{cF_c}{\gamma H}$

Key: (C)

Sol: As per Taylors stability number method

$$\text{Factor of safety}\,(F_c)=\frac{C}{S_n\gamma H}\Rightarrow S_n=\frac{C}{\gamma F_c H}$$

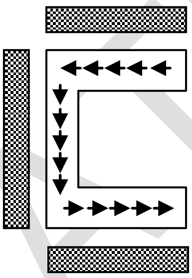
9. A closed thin walled tube has thickness, t, mean enclosed area within the boundary of the centerline of tube's thickness, A_m, and shear stress τ. Torsional moment of resistance, T of the section would be

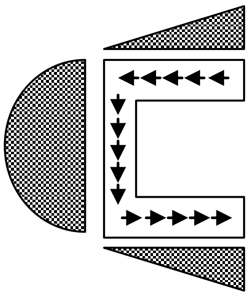
(A) $2\tau A_m t$ (B) $\tau A_m t$ (C) $0.5\tau A_m t$ (D) $4\tau A_m t$

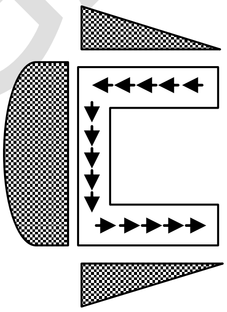
Key: (A)

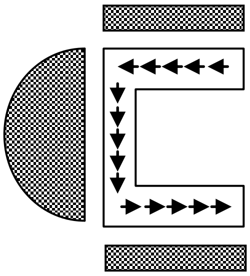
Sol: Shear stress $\tau=\frac{T}{2A_m t}\Rightarrow T=2A_m\tau t$

10. For a channel section subjected to a downward vertical shear force at its centroid, which one of the following represents the correct distribution of shear stress in flange and web?

(A)


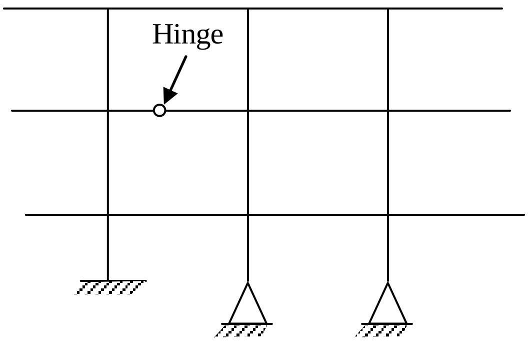
(B)


(C)


(D)


Key: (C)

11. The degree of static indeterminacy of the plane frame is shown in the figure is____



Key: (15)

Sol: Number of support reactions, (r) $=3+2+2=7$

Number of members (m) = 21

Number of closed loops(C)=4

Degree of static indeterminacy $(D_s) = D_{si} + D_{se} = 3C + (r-3)$

$$= 3\times 4 + (7-3) = 12+4 = 16$$

Force releases = 1 at the Hinge $D_s = 16-1 = 15$

12. Structural failures considered in the mechanistic method of bituminous pavement design are

(A) Fatique and shear

(B) Rutting and shear

(C) Shear and slippage

(D) Fatigue and Shear

Key: (D)

Sol: **As** per IRC, Fatigue and shear failure criteria's can be considered for the design of bituminous pavement.

13. Which one of the options contains ONLY primary air pollutants?

(A) Hydrocarbons and nitrogen oxides

(B) Nitrogen oxides and peroxyacetyl nitrate

(C) Ozone and peroxyacetyl nitrate

(D) Hydrocarbons and ozone

Key: (A)

Sol: Based on origin, air pollutants are classified into two categories

(i) Primary air pollutants: Particulates, CO, SO_x, NO_x, hydrocarbons …….etc.

(ii) Secondary air pollutants: Ozone, PAN, PBN, PPN, Smog…. etc.

14. The following inequality is true for all x close to 0.

$$2-\frac{x^2}{3}<\frac{x\sin x}{1-\cos x}<2$$

What is the value of $\lim_{x\to 0}\frac{x\sin x}{1-\cos x}$?

(A) 2 (B) 1/2 (C) 0 (D) 1

Key: (A)

Sol: $\lim_{x\to 0}\frac{x\sin x}{1-\cos x}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{x\cos x+\sin x}{+\sin x}\left(\frac{0}{0}\right)\left[\text{u sin g Lhorpital rule}\right]$

$$=\lim_{x\to 0}\frac{x(-\sin n)+\cos x+\cos x}{+\cos x}\left[\text{Using L'Horpital rule}\right]$$

$$=\frac{0+1+1}{1}=\frac{2}{1}=2.$$

15. The velocity field in a flow system is given by $v=2i+(x+y)j+(xyz)k$. The acceleration of the fluid at (1, 1, 2) is

(A) j + k (B) 2i + 10 k (C) 4j + 10k (D) 4i + 12 k

Key: (C)

Sol: Velocity field $(v)=2i+(x+y)j+(xyz)k$

$$\text{Acceleration } (a_n)=\frac{d(v)}{dt}=\underbrace{\left(\frac{\partial v}{\partial t}\right)}_{\text{Temporal acceleration}}+\underbrace{\left(u\frac{\partial v_n}{\partial x}+v.\frac{\partial v_n}{\partial y}+w\right)}_{\text{Convective acceleration}}$$

Here, $u=2;\quad v=x+y;\quad w=xyz;\quad t=0;\quad n \text{ is direction}$

$$a_x=0+2\times\frac{\partial}{\partial x}(2)+(x+y).\frac{\partial(2)}{\partial y}+(xyz)\frac{\partial(2)}{\partial z}$$

$$a_x=0$$

$$a_y=0+2.\frac{\partial}{\partial w}(x+y)+(x+y)\frac{\partial}{\partial y}(x+y)+(xyz)\frac{\partial}{\partial z}(x+y)$$

$$a_y=2+x+y$$

$$a_z=0+2\frac{\partial}{\partial x}(xyz)+(x+y)\frac{\partial}{\partial y}(xyz)+xyz.\frac{\partial}{\partial z}(xyz)$$

$$=0+2.yz+(x+y)(xz)+xyz\ xy$$

$$a_z=2yz+(x+y)(xz)+x^2y^2z$$

$$\text{Acceleration }=a_xi+a_yj+a_zk$$

$$a=0+(2+x+y)j+\left(2yz+(x+y)xz+x^2y^2z\right)k$$

$$a_{(1,1,2)} = 0 + (2+1+1)j\left(2\times1\times2 + (1+1)1\times2 + 1^2\times1^2\times2\right)k$$
$$= 4j + (4+4+2)k = 4j + 10k$$

16. A steel column is restrained against both translation and rotation at one end and is restrained only against rotation but free to translate at the other end. Theoretical and design (IS: 800 – 2007) values, respectively, of effective length factor of the column are

(A) 1.0 and 1.0 (B) 1.2 and 1.0 (C) 1.0 and 1.2 (D) 1.2 and 1.2

Key: (C)

Sol: As per IS:800-2007

For column, restrained against both rotation and transalation at one end and restrained only against rotation but free to translate at other end

Theoretical value of effective length factor = 1.0

Design value of effective length factor = 1.2

17. Euclindean norm (length) of the vector $[4–2\text{-}6]^T$ is

(A) $\sqrt{48}$ (B) $\sqrt{24}$ (C) $\sqrt{12}$ (D) $\sqrt{56}$

Key: (D)

Sol: Given, vector is

$$[4-2-6]^T = \begin{bmatrix} 4 \\ -2 \\ -6 \end{bmatrix}$$

$$\therefore \text{length of the vector} = \sqrt{4^2 + (-2)^2 + (-6)^2} = \sqrt{16+4+36} = \sqrt{56}$$

18. An inflow hydrograph is routed through a reservoir to produce an outflow hydrograph. The peak flow of the inflow hydrograph is P_I and the time of occurrence of the peak is t_I. The peak flow of the outflow hydrograph is P_0 and the time of occurance of the peak is t_0.

Which one of the following statements is correct?

(A) $P_I < P_0$ and $t_I > t_0$ (B) $P_I > P_0$ and $t_I > t_0$

(C) $P_I < P_0$ and $t_I < t_0$ (D) $P_I > P_0$ and $t_I < t_0$

Key: (D)

19. Construction of a new building founded on a clayey soil was completed in January 2010. In January 2014, the average consolidation settlement of the foundation in clay was recorded as 10

mm. The ultimate consolidation settlement was estimated in design as 40 mm. Considering double drainage to occur at the clayey soil site, the expected consolidation settlement in January 2019 (in mm, round off to the nearest integer) will be_________.

Key: (15)

Sol: **January-2014**

Time=4years

Consolidation settlement (S) = 10mm

Ultimate settlement (S_f) = 40mm

Degree of consolidation (U) $= \frac{S}{S_f} \times 100 = \frac{10}{40} \times 100 = 25\%$

January-2019

Time = 9years

Drainage is double drainage

$$U \le 60 \Rightarrow (T_v)_1 = \frac{\pi}{4}(U)^2 = (T_v)_1 = \frac{\pi}{4}(0.25)^2 = 0.0491$$

$$T_v = \frac{C_v.t}{H^2} = \frac{\pi}{4}U^2 \Rightarrow U^2 \propto t$$

$$\frac{U_1^2}{U_2^2} = \frac{t_1}{t_2} = \frac{(0.25)^2}{U_2^2} = \frac{4}{9} \Rightarrow U_2^2 = \frac{9}{4} \times 0.25^2$$

$$U_2 = \sqrt{\frac{9}{4} \times 0.25^2} = 0.375$$

Total consolidation = $40 \times 0.375 = 15mm$

20. The notation "SC" as per Indian standard Soil Classification System refers to

(A) Clayey silt (B) Sandy clay (C) Clayey sand (D) Silty clay

Key: (C)

21. The speed-density relationship in a mid-block section of a highway follows the Greenshield's model. If the free flow speed is v_f and the jam density is k_j, the maximum flow observed on this section is

(A) $\frac{v_f k_j}{2}$ (B) $\frac{v_f k_j}{8}$ (C) $v_f k_j$ (D) $\frac{v_f k_j}{4}$

Key: (D)

Exp: Maximum capacity $(q_{max}) = V_{max}.K_{max}$

As per Greenshield's model

V_{max} will occur at $\frac{V_{freemean}}{2}$

K_{max} will occur at $\frac{K_{Jam}}{2}$

$$\text{Maximum capacity } (q_{mean}) = \frac{V_f}{2}\frac{k_J}{2} = \frac{V_f k_J}{4}$$

22. An anisotropic soil deposit has coefficient of permeability in vertical and horizontal directions as k_z and k_x, respectively. For constructing a flow net, the horizontal dimension of the problem's geometry is transformed by a multiplying factor of

(A) $\frac{k_z}{k_x}$ (B) $\frac{k_x}{k_z}$ (C) $\sqrt{\frac{k_z}{k_x}}$ (D) $\sqrt{\frac{k_x}{k_z}}$

Key: (C)

23. The Laplace transform of sinh (at) is

(A) $\frac{s}{s^2-a^2}$ (B) $\frac{s}{s^2+a^2}$ (C) $\frac{a}{s^2-a^2}$ (D) $\frac{a}{s^2+a^2}$

Key: (C)

Sol: $L[\sin h(at)] = \frac{a}{s^2-a^2}$, using definition of L.T.

24. The data from a closed traverse survey PQRS (run in the clockwise direction) are given in the table

Line	Included angle (in degree)
PQ	88
QR	92
RS	94
SP	89

The closing error for the traverse PQRS (in degrees) is_______.

Key: (3)

Sol: In a closed traverse

$$\text{Sum of interior angles} = (2n-4)\times 90 = (2\times 4-4)\times 90 = 360 \text{ degrees}$$
$$\text{The sum of angles given} = 88+92+94+89 = 363 \text{ degrees}$$
$$\text{Error} = 363-360 = 3 \text{ degrees}$$

25. A vehicle is moving on a road of grade +4% at a speed of 20 m/s. Consider the coefficient of rolling friction as 0.46 and acceleration due to gravity as 10 m/s^2. On applying brakes to reach a speed of 10 m/s, the required braking distance (in m, round off to nearest integer) along the horizontal, is ______.

Key: (30)

Sol: Initial speed (V_i)=20m/sec

Coefficient of friction (μ) = 0.46

Final speed (V_f) = 10m/sec

$$\text{Braking distance } = \frac{V_i^2}{2g(\mu+0.01n)} - \frac{V_f^2}{2g(\mu+0.01n)}$$

$$= \frac{V_i^2 - V_f^2}{2g(\mu+0.01n)} = \frac{20^2 - 10^2}{2\times 10(0.46+0.01\times 4)} = 30\text{m}$$

Q. No. 26 - 55 Carry Two Marks Each

26. A broad gauge railway line passes through a horizontal curved section (radius = 875 m) of length 200 m. The allowable speed on this portion is 100 km/h. For calculating the cant consider the gauge as centre-to-centre distance between the rail heads, equal to 1750mm, The maximum permissible cant (in mm, round of to 1 decimal place) with respect to the centre-to-centre distance between the rail heads is ______.

Key: (157.5)

Sol: For a railway track

$$\text{Allowable cant} = \frac{G.V^2}{127R} \quad \text{where V is in km/hr } = 100$$

$$\text{Allowable cant } = \frac{1750\times 100^2}{127\times 895} = 157.5\,\text{mm}$$

27. When a specimen of M25 concrete is loaded to a stress level of 12.5 MPa, a strain of 500×10^{-6} is recorded. If this load is allowed to stand for a long time, the strain increases to 1000×10^{-6}. In accordance with provisions of IS: 456-2000, considering the long-term effects, the effective modulus of elasticity of the concrete (in MPa) is ________

Key: (12500)

Sol: Effective modulus (or) long term modulus $(E_\theta) = \dfrac{E_C}{1+\theta}$

$$E_C = 5000\sqrt{f_{ck}} = 5000\sqrt{25} = 250000\text{MPa}$$

$$\text{creep coefficient}(\theta) = \frac{1000\times10^{-6} - 500\times10^{-6}}{500\times10^{-6}} = 1$$

$$E_\theta = \frac{250000}{1+1} = \frac{25000}{2} = 12500\text{MPa.}$$

28. The probability density function of a continuous random variable distributed uniformly between x and y (for y > x) is

(A) $\frac{1}{x-y}$ (B) $x-y$ (C) $y-x$ (D) $\frac{1}{y-x}$

Key: (D)

Sol: The probability density function

$$f(x)=\frac{1}{y-x} \text{ for } y>x.$$

[Using uniform distribution over [x, y]]

29. The uniform arrival and uniform service rates observed on an approach road to a signalized intersection are 20 and 50 vehicles/minutes, respectively. For this signal, the red time is 30 s, the effective green time is 30 s, and the cycle length is 60s. Assuming that initially there are no vehicles in the queue, the average delay per vehicle using the approach road during a cycle length (in s, round off to 2 decimal places) is ________ .

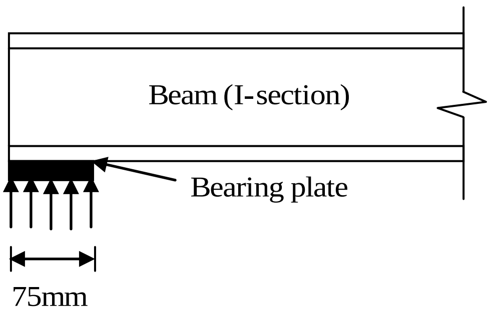
Key: (12.5)

Sol: Average delay $(d_i)=\dfrac{\dfrac{C_o}{2}\left(1-\dfrac{G_i}{C_o}\right)^2}{1-\dfrac{q_i}{S_i}}$

Cycle length (C_o) = 60 seconds
Normal flow (q_i) = 20 veh/min
Saturation flow (S_i) = 50 veh/min

$$d_i=\frac{\dfrac{60}{2}\left[1-\dfrac{30}{60}\right]}{1-20/50}=12.5\text{seconds}$$

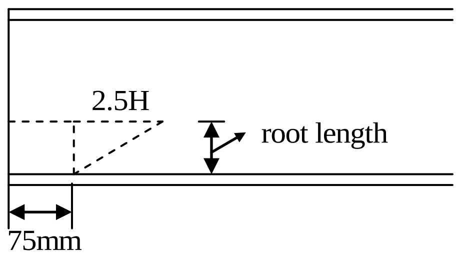
30. A rolled I-section beam is supported on a 75 mm wide bearing plate as shown in the figure. Thickness of flange and web of the 1-section are 20 mm and 8mm, respectively. Root radius of the I-section is 10mm. Assuming: material yield stress, f_y = 250 MPa and partial safety factor for material, $\gamma_{mo}=1.10$



As per IS: 800-2007, the web bearing strength (in kN, round off to 2 decimal places) of the beam is ________

Key: (272.23)

Sol:



Effective are in bearing = [75 + 2.5(flange thickness + root length)] × 8

= [75 + 2.5(20 + 10)] × 8 = 1200 mm^2

So, web bearing strength $= A_g \frac{f_y}{1.1} = 1200 \times \frac{250}{1.1} \times 10^{-3}$

$= 272.73\,\text{kN}$

31. The critical bending compressive stress in the extreme fibre of a structural steel section is 1000 MPa. It is given that the yield strength of the steel is 250 MPa, width of flange is 250 mm and thickness of flange is 15 mm. As per the provisions of IS: 800-2007, the nondimensional slenderness ratio of the steel crosssection is

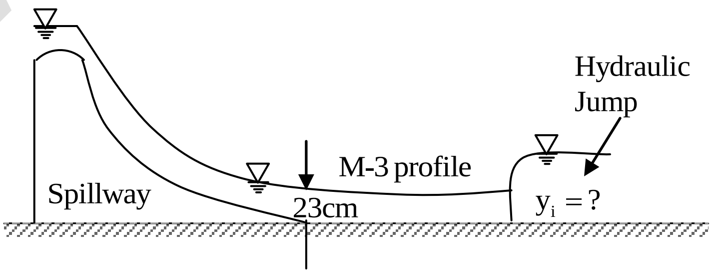
(A) 0.50 (B) 2.00 (C) 0.25 (D) 0.75

Key: (A)

Sol: As per IS800 : 2007

Non-dimensional slenderness ratio $(\lambda) = \sqrt{\frac{f_y}{f_{cr}}} = \sqrt{\frac{250}{1000}} = 0.5$

32. At the foot of a spillway, water flows at a depth of 23 cm with a velocity of 8.1 m/s, as shown in the figure.

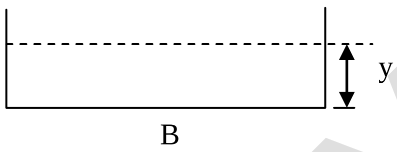


The flow enters as an M-3 profile in the long wide rectangular channel with bed slope $\frac{1}{1800}$ and Manning's n = 0.015. A hydraulic jump is formed at a certain distance from the foot of the spillway. Assume the acceleration due to gravity, $g = 9.81\ m/s^2$. Just before the hydraulic jump, the depth of flow y_1 (in m, round off to 2 decimal places) is ________.

Key: (0.42)

Sol: Wide rectangular channel (B>>y)

$$A = B \times y$$
$$P = B + 2y$$
$$R = \frac{A}{P} = \frac{By}{B+2y} = \frac{By}{B} = y$$



Given profile is $M_3 \Rightarrow NDL > CDL$

By manning's equation

$$Q = \frac{1}{n}.A.R^{2/3}.S^{1/2}$$
$$Q = \frac{1}{n}.(By).y^{2/3}.S^{1/2}$$
$$\frac{Q}{B} = \frac{1}{n}.y^{5/3}S^{1/2}$$
$$\frac{AV}{B} = \frac{1}{n}.y^{5/3}S^{1/2}$$
$$\frac{B.y\,V}{B} = \frac{1}{n}.y^{5/3}.S^{1/2}$$
$$0.23 \times 8.1 = \frac{1}{0.015} \times y_n^{5/3}\left(\frac{1}{1800}\right)^{1/2}$$
$$y_n = 1.107m$$

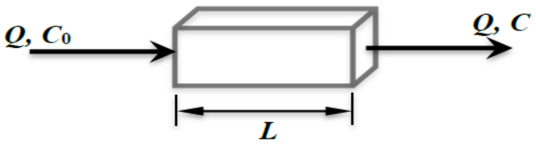
Before the hydraullic jump, depth = y_1

y_n is the conjugate depth of y_1

$$\frac{y_1}{y_n} = \frac{-1}{2}\left[1 - \sqrt{1 + 8.F_n^2}\right]$$
$$y_1 = \frac{-y_n}{2}\left[1 - \sqrt{1 + 8 \times \frac{q^2}{g.y^3}}\right]$$
$$y_1 = \frac{-1.107}{2}\left[1 - \sqrt{1 + \frac{8 \times 1.863^2}{9.81 \times 1.107^3}}\right] = 0.42m$$
$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.863^2}{9.81}\right)^{1/3} = 0707m.$$

33. Consider the reactor shown in the figure. The flow rate through the reactor is Q m^3/h .The concentration (in mg/L) of a compound in the influent and effluents are C_0 and C, respectively. The compound is degraded in the reactor following the first order reactions. The mixing

condition of the reactor can be varied such that the reactor becomes either a completely mixed flow reactor (CMFR) or a plug-flow reactor (PFR). The length of the reactor can be adjusted in these two mixing conditions to L_{CMFR} and L_{PER} while keeping the cross-section of the reactor constant. Assuming steady state and for C/C_0 = 0.8, the value of L_{CMFR}/L_{PER} (round off to 2 decimal places) is _______



Key: (1.12)

Sol: Given $\frac{C}{C_0} = 0.8$

For CMFR	For PFR
$\frac{C}{C_0} = \frac{1}{1+K\theta}$	$\frac{C}{C_0} = e^{-K\theta}$
$0.8 = \frac{1}{1+K\theta_1}$	$e^{-k\theta_2} = 0.8$
$0.8 + 0.8K\theta_1 = 1$	$-K\theta_2 = -0.2231$
$0.8K\theta_1 = 0.2$	$K\theta_2 = 0.2231$
$K\theta_1 = 0.25$	

$$\frac{K\theta_1}{K\theta_2} = \frac{K \times \frac{V_1}{Q}}{K \times \frac{V_2}{Q}} = \frac{V_1}{V_2} = \frac{A_1 \times L_1}{A_2 \times L_2} = \frac{0.25}{0.2231} = 1.12$$

34. A series of perpendicular offsets taken from a curved boundary wall to a straight survey line at an interval of 6 m are 1.22, 1.67, 2.04, 2.34, 2.14, 1.87, and 1.15 m. The area (in m^2, round off to 2 decimal places) boundary by the survey line, curved boundary wall, the first and the last offsets, determined using Simpson's rule, is _______.

Key: (68.56)

Sol: Using simpson's rule, we have

$$\int_{x_0}^{x_n} f(x)dx \approx \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + ... + y_{n-2}) + 4(y_1 + y_3 + ... + y_{n-1})\right] \quad ...(1)$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6
$y = f(x)$	1.22	1.67	2.04	2.34	2.14	1.87	1.15
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Wher 'y' denotes perpendicular off sets taken from a curved boundary wall to a straight survey line [x-axis].

Required Area [from (1)];

$$\int_{x_0}^{x_6} f(x)dx \approx \frac{6}{3}\left[\left(\underset{y_0}{\underset{\downarrow}{1.22}}+\underset{y_6}{\underset{\downarrow}{1.15}}\right)+2\left[2.04+2.14\right]+4\left[1.67+2.34+1.87\right]\right]; \quad \text{where } h = 6.$$

$$= 2\left[2.37+2(4.18)+4[5.88]\right]$$
$$= 2\left[2.37+8.36+23.52\right] = 68.5m^2.$$

35. A water treatment plant treats 6000 m^3 of water per day. As a part of the treatment process, discrete particles are required to be settled in a clarifier. A column test indicates that an overflow rate of 1.5 m per hour would produce the desired removal of particles through settling in the clarifier having a depth of 3.0 m. The volume of the required clarifier, (in m^3, round off to 1 decimal place) would be

Key: (500)

Sol: $\text{Flow}(Q) = 6000m^3/\text{day} = \frac{6000}{24} m^3/hr = 250m^3/h$

$\text{Overflow rate}(V_o) = 1.5m^3/h$

$\text{Area} = \frac{Q}{V_o} = \frac{250}{1.5} = 166.67m^2$

$\text{Volume} = \text{Area} \times \text{depth} = \frac{250}{1.5} \times 3 = 500m^3$

36. A flexible pavement has the following class of loads during a particular hour of the day.

(i) 80 buses with 2-axles (each axle load of 40 kN);

(ii) 160 trucks with 2-axles (front and rear axle loads of 40 kN and 80 kN, respectively)

The equivalent standard axle load repetitions for this vehicle combination as per IRC:37-2012 would be

(A) 320 (B) 250 (C) 240 (D) 180

Key: (D)

Sol: Standard axle load = 80kN

$$\text{Equivalent standard axle load} = 280 \times \left(\frac{40}{80}\right)^4 + 160 \times \left(\frac{40}{80}\right)^4 + 160 \times \left(\frac{80}{80}\right)^4$$
$$= 160 \times \left(\frac{1}{2}\right)^4 + 160 \times \left(\frac{1}{2}\right)^4 + 160 \times 1^4$$
$$= 20 + 160 = 180$$

37. Constant head permeability tests were performed on two soil specimens, S1 and S2. The ratio of height of the two specimens (L_{S1}: L_{S2}) is 1.5, the ratio of the diameter of specimens (DS1:DS2) is 0.5, and the ratio of the constant head (h_{s1}:h_{s2}) applied on the specimens is 2.0. If the discharge from both the specimens is equal, the ratio of the permeability of the soil specimens (k_{s1}:k_{s2}) is ________

Key: (3)

Sol: $\frac{(L_s)_1}{(L_s)_2}=1.5\,;\ \frac{(D_s)_1}{(D_s)_2}=0.5,\ \frac{(h_s)_1}{(h_s)_2}=2.0$

By constant head permeability test

$$Q = k\,i\,A$$

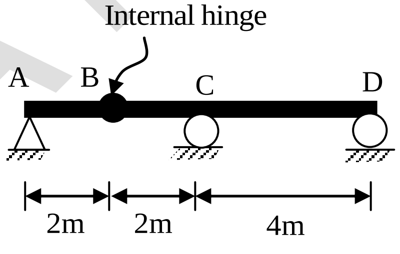
$$Q = k\times\frac{h}{L}\times A = \frac{kh}{L}\frac{\pi}{4}\times d^2 \Rightarrow Q \propto \frac{kh}{L}d^2$$

$$\frac{Q_1}{Q_2} = \frac{\frac{k_1h_1}{L_1}d_1^2}{k_2\frac{h_2}{L_2}d_2^2} = \frac{k_1}{k_2}\times\frac{h_1}{h_2}\times\frac{1}{\left(\frac{L_1}{L_2}\right)}\left(\frac{d_1}{d_2}\right)^2$$

$$\text{As}\quad Q_1 = Q_2 \ \Rightarrow 1 = \frac{k_1}{k_2}\times 2\times\frac{1}{1.5}\times(0.5)^2$$

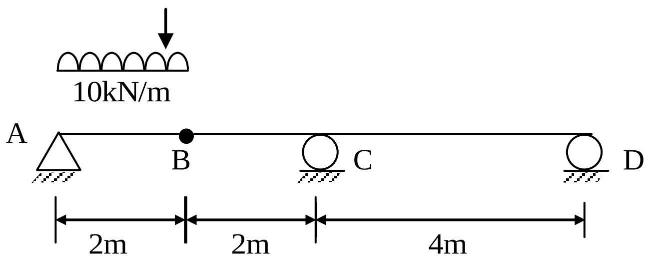
$$\frac{k_1}{k_2} = \frac{1.1.5}{2\times 0.5^2} = 3$$

38. A long uniformly distributed load of 10 kN/m and a concentrated load of 60 kN are moving together on the beam ABCD shown in the figure (not drawn to scale). The relative positions of the two loads are not fixed. The maximum shear force (in kN, round off to the nearest integer) caused at the internal hinge B due to the two loads is _______



Key: (70)

Sol:



Influence line for shear force at B.

From muller's breslauce principle

$$\text{Maximum shear force} = 60\times1+\left(\frac{1}{2}\times1\times2\right)\times10$$
$$= 60+10=70\text{kN}$$

39. A square footing of 2m sides rests on the surface of a homogeneous soil bed having the properties: cohesion c = 24 kPa, angle of internal friction $\phi=25°$, and unit weight $\gamma=18\text{kN/m}^3$. Terzaghi's bearing capacity factor $\phi=25°$ are Nc = 25.1, Nq = 12.7, $N_\gamma=9.7$, $N'_c=14.8$, $N'_q=5.6$ and $N'_\gamma=3.2$. The ultimate bearing capacity of the foundation (in kPa, round off to 2 decimal places) is ______.

Key: (353.92)

Sol: $\phi=25°<29° \Rightarrow$ Local shear failure

$$C_m=\frac{2}{3}C=\frac{2}{3}\times24=16\text{kPa}$$

As per Terraghi's theory

For square footing

$$\text{Ultimate bearing capacity } (q_u)=1.3C_mN_C+\gamma D_f N_q+0.4\gamma B.\ N_\gamma$$
$$=1.3\times16\times14.8+0+0.4\times18\times2\times3.2$$
$$=307.84+46.08=353.92\text{kPa}$$

40. The dimensions of a soil sampler are given in the tale.

Parameter	**Cutting edge**	**Sampling tube**
Inside diameter (mm)	80	86
Outside diameter (mm)	100	90

For this sampler, the outside clearance ratio (in percent, round off to 2 decimal places) is ________.

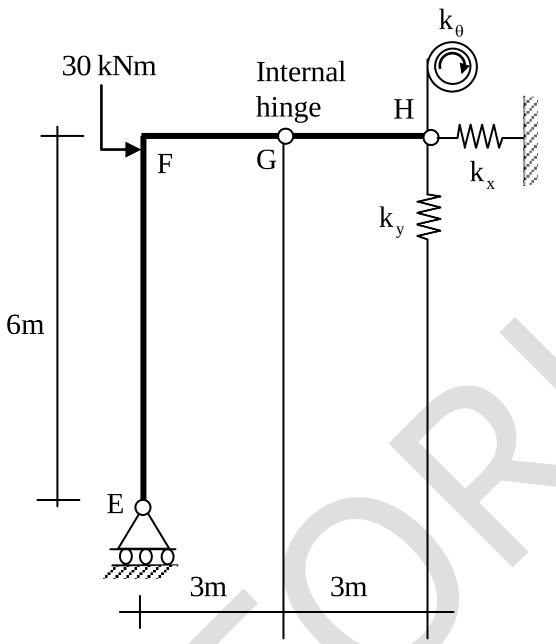
Key: (11.11)

Sol: $D_1=80\text{mm}$ $D_3=86\text{mm}$

$D_2=100\text{mm}$ $D_4=90\text{mm}$

Outside clearance ratio $= \dfrac{D_2 - D_4}{D_4} \times 100 = \dfrac{100 - 90}{90} \times 100 = 11.11\%$

41. A plane frame shown in the figure (not to scale) has linear elastic springs at node H. The spring constants are $k_x = k_y = 5 \times 10^5$ kN/m^3 and $k_\theta = 3 \times 10^5$ kNm/rad.



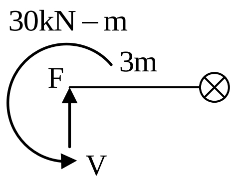
For the externally applied moment of 30 kNm at node F, the rotation (in degrees, round off to 3 decimals) observed in the rotational spring at node H is ________.

Key: (0.006)

Sol: Since horizontal reaction at $E = H_c = 0$

Bending moment at F for member $FE = M_{FE} = 0$

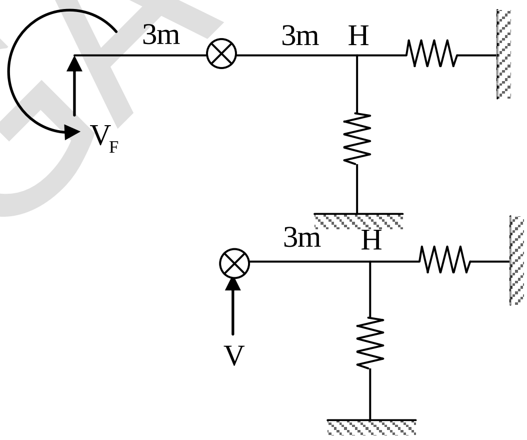
Member FH behaves like propped cantilever beam



FBD of FG

$\Sigma m_F = 0$

$v = 10$ kN



FBD of FG

$v \times 3 = k_\theta \times \theta$

$$M_H = \frac{F_{BD}}{V \times 3} = \frac{GH}{K_0 \times \theta}$$
$$\theta = \frac{10 \times 3}{3 \times 10^5} = 10^{-4} \text{rad} = 0.0057 \text{ deg}$$

42. Chlorine is used as the disinfectant in a municipal water treatment plant. It achieves 50 percent of disinfection efficiency measured in terms of killing the indicator microorganisms (E-Coli) in 3 minutes. The minimum time required to achieve 99 percent disinfection efficiency would be

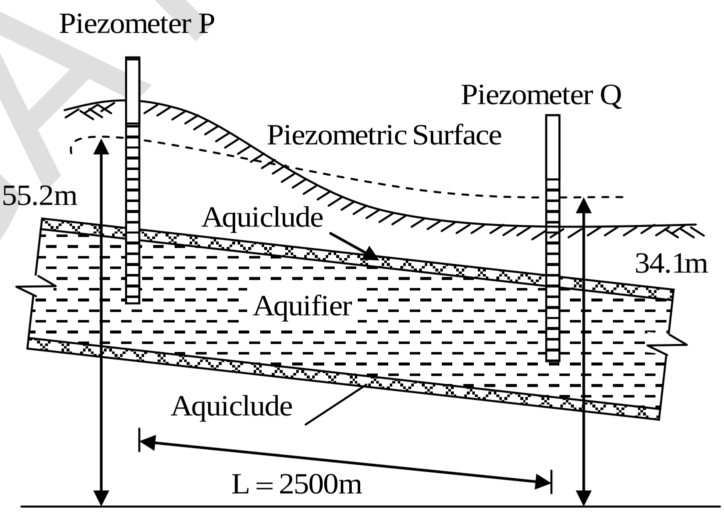
(A) 9.93 minutes
(B) 11.93 minutes
(C) 21.93 minutes
(D) 19.93 minutes

Key: (D)

Sol: Disinfection efficiency $\eta_1 = 50\%$

$$\eta = 1 - e^{-k.t} \Rightarrow 0.50 = 1 - e^{-k \times 3}$$
$$e^{-3k} = 0.5$$
$$\Rightarrow -3k = \ell n 0.5 \Rightarrow -3k = -0.693 \Rightarrow k = 0.23/mm$$
$$\text{For } \eta = 99\% \Rightarrow \eta = 1 - e^{-kt}$$
$$0.99 = 1 - e^{-0.23 \times t} \Rightarrow e^{-0.23t} = 0.01$$
$$\Rightarrow -0.23t = \ell n(0.01) \Rightarrow -0.23t = -4.60$$
$$\therefore t = 20 \text{minutes.}$$

43. A confined aquifer of 15 m constant thickness is sandwiched between two aquicludes as shown in the figure (not drawn to scale)



The heads indicated by two piezometers P and Q are 55.2 m and 34.1 m, respectively. The aquifer has a hydraulic conductivity of 80 m/day and its effective porosity is 0.25. If the distance between the piezometers is 2500 m, the time taken by the water to travel through the aquifer from piezometer location P to Q (in days, round off to 1 decimal place) is ______.

Key: (925.65)

Sol: From the figure

$$\Delta h = 55.2 - 34.1 = 21.1\text{m}$$

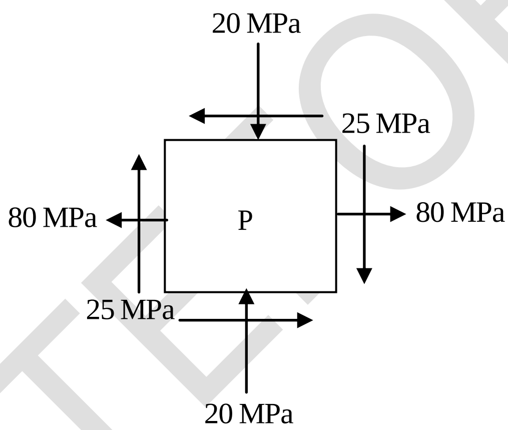
$$\text{time taken} = \frac{\text{Length}}{\text{Seepage velocity}}$$

$$\text{Seepage velocity}(V_s) = \frac{\text{Velocity}}{\text{Porosity}} = \frac{Ki}{n} = \frac{k.\frac{\Delta h}{L}}{n} = \frac{k\Delta h}{Ln}$$

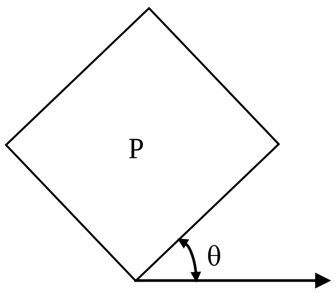
$$V_s = \frac{80\text{m/day} \times 21.1\text{m}}{2500\text{m} \times 0.25} = 2.7008\text{m/day}$$

$$\text{Time taken} = \frac{1}{V_s} = \frac{2500\text{m}}{2.7008\text{m/day}} = 925.65\,\text{days}$$

44. For a plane stress problem, the state of stress at a point P is represented by the stress element as shown in the figure.



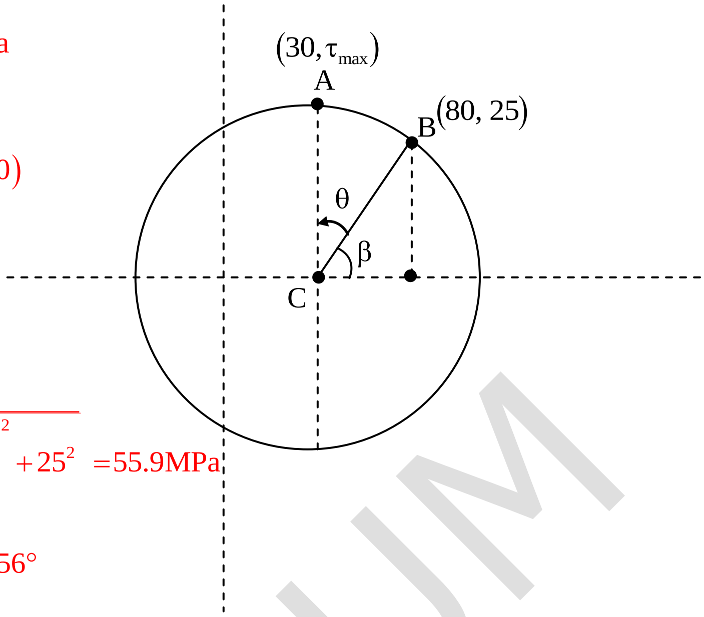
By how much angle (θ) in degrees the stress element should be rotated in order to get the planes of maximum shear stress?



(A) 26.6 (B) 48.3 (C) 31.7 (D) 13.3

Key: (C)

Sol: $\sigma_x = 80\text{MPa},\ \tau = 25\text{MPa},\ \sigma_y = 20\text{MPa}$



Centre of Mohr's circle

$$= \left(\frac{\sigma_x + \sigma_y}{z},\ 0\right) = \left(\frac{80-20}{2}, 0\right) = (30,\ 0)$$

Radius of Mohr's circle ,

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{80-(-20)}{2}\right)^2 + 25^2} = 55.9\text{MPa}$$

$$\text{Tan}\beta = \frac{25}{80-30} \Rightarrow \beta = \text{Tan}^{-1}(0.5) = 26.56°$$

$$\theta = 90 - \beta = 90 - 26.56 = 63.44°$$

$$\text{Angle to be rotaed} = \frac{63.44}{2} = 31.7°$$

45. The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} 2 & -\frac{4}{5} & -\frac{9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & -\frac{1}{5} & -\frac{6}{5} \end{bmatrix}$

(B) $\begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

(C) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

(D) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

Key: (B)

Sol: Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$$\therefore \text{Adj}(A) = \begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}^T \ \&$$

$$|A| = \begin{vmatrix} + & - & + \\ 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{vmatrix} = 2[12-2]-3[16-1]+4[8-3]$$

$$\Rightarrow |A| = 20 - 45 + 20 = -5$$

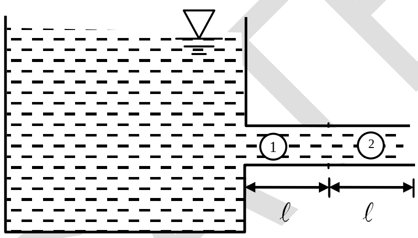
$$\therefore A^{-1} = \frac{\text{Adj}(A)}{|A|} = -\frac{1}{5}\begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}^T$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ \frac{4}{5} & \frac{-4}{5} & \frac{1}{5} \\ \frac{9}{5} & \frac{14}{5} & \frac{6}{5} \end{bmatrix}^T = \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & \frac{-4}{5} & \frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$$

46. Two identical pipes (i.e. having the same length, same diameter,and same roughness) are used to withdraw water from a reservoir. In the first case, they are attached in series and also discharge freely into the atmosphere. In the second case, they are attached in parallel and friction factor is same in both the cases, the ratio of the discharge in the parallel arrangement to that in the series arrangement (round off to 2 decimal places) is _____

Key: (2.83)

Sol: 1^{st} case: Series connecton



For series connection

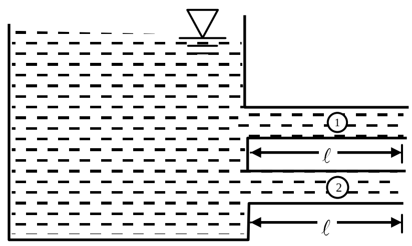
total head loss due to friction

$$h_f = h_{f_1} + h_{f_2}$$

$$h_f = \frac{f_1 L_1 Q_1^2}{12.1 d_1^5} + \frac{f_2 L_2 Q_2^2}{12.1 d_2^5}$$

$$\Rightarrow (h_{f_1}) = \frac{2fLQ_I^2}{12.1 d^5}$$

2^{nd} case: Parallel connection



For parallel connection

total head loss due to friction

is same in both pipes $Q_1 = Q_2 = \frac{Q}{2}$

$$h_f = \frac{fL\left(\frac{Q}{2}\right)^2}{12.1 d^5} = \frac{fLQ_{II}^2}{4 \times 12 \times d^5}$$

$$\text{as given } (h_f)_2 = (h_f)_1$$

$$\frac{(h_f)_2}{(h_f)_1} = \frac{\frac{Q_{II}^2}{4}}{2Q_I^2} \Rightarrow \left(\frac{Q_{II}}{Q_I}\right)^2 = 8 \Rightarrow \frac{Q_{II}}{Q_I} = \sqrt{8} = 2.83$$

47. A camera with a focal length of 20 cm fitted in an aircraft is used for taking vertical aerial photographs of a terrain. The average elevation of the terrain is 1200 m above mean sea level (MSL). What is the height above MSL at which an aircraft must fly in order to get the aerial photographs at a scale of 1:8000?

(A) 3200 m (B) 2800 m (C) 2600 m (D) 3000 m

Key: (B)

Sol: We know that

Scale of photograph $S=\dfrac{f}{H-h}$

$$\frac{1}{8000}=\frac{0.2}{H-1200}$$

$$H-1200=8000\times 0.2$$

$$H-1200=1600=2800m$$

48. Raw municipal solid waste (MSW) collected from a city contains 70% decomposable material that can be converted to methane. The water content of the decomposable material is 35%. An elemental anlysis of the decomposable material yields the following mass percent.

C:H:O:N: other = 44:6:43:0.8:6.2

The methane production of the decomposable material is governed by the following stoichiometric relation

$$C_aH_bO_cN_d+nH_2O\rightarrow mCH_4+sCO_2+dNH_3$$

Given atomic weight: C = 12, H = 1, O = 16, N = 14. The mass of methane produced (in grams, round off to 1 decimal place) per kg of raw MSW will be _________.

Key: (137.6)

Sol: Let 1 kg of raw municipal solid waste (MSW)

Decomposable material = 70% of 1 kg = 0.7 kg

Water content = 35%

$$\text{Weight of solids in MSW}=\left(\frac{100-35}{100}\right)\times 0.7=0.455\text{ kg}=455\text{gms}$$

Given: C:H:O:N: other ≡ 44 : 6: 43: 0.8:6.2

$$\text{Weight of }C=\frac{(44)}{(44+6+43+0.8+6.2)}\times 455=200.2\text{ gms}$$

$$\text{Weight of }H=\frac{(6)}{(44+6+43+0.8+6.2)}\times 455=27.3\text{ gms}$$

$$\text{Weight of }O=\frac{(43)}{(44+6+43+0.8+6.2)}\times 455=195.6\text{ gms}$$

$$\text{Weight of } N = \frac{(0.8)}{(44+6+43+0.8+6.2)} \times 455 = 3.64 \text{ gms}$$

$$\Rightarrow 12a = 200.2 \Rightarrow a = 16.68$$
$$1 \times b = 27.3 \Rightarrow b = 27.3$$
$$16 \times c = 195.6 \Rightarrow c = 12.225$$
$$14 \times d = 3.64 \Rightarrow d = 0.26$$

Now, balancing the given stoichiometric $r \times n$

$$C_aH_bO_CN_d + nH_2O \rightarrow mCH_4 + SCO_2 + dNn_3$$

$$12a = 12m + 12s \Rightarrow 12m + 12s = 200.2 \qquad (12a = 200.2)$$
$$b + 2n = 4m + 3d \Rightarrow 4m - 2n = 26.52 \qquad (6 = 27.3)$$
$$16c + 16n = 325 \Rightarrow n = 25 - 12.225 \qquad (16c = 195.6)$$

$$\therefore \quad 4m - 2(2s - 12.225) = 26.52$$

$$\Rightarrow 4m - 4s = 2.07$$
$$\Rightarrow 12m - 12s = 6.21$$
$$\text{Also, } 12m + 12s = 200.2$$
$$\Rightarrow m = 8.6$$
$$\therefore \text{ Mass of methane produced } = 16 \times m = 16 \times 8.6 = 137.6 \text{ gms}$$

49. In the context of provisions relating to durability of concrete, consider the following assertion:

Assertion (1): As per IS 456-2000, air entrainment to the extent of 3% to 6% is required for concrete exposed to marine environment.

Assertion (2): The equivalent alkali content (in terms of Na_2O equivalent) for a cement containing 1% and 0.6% of Na_2O and K_2O respectively, is approximately 1.4% (rounded to 1 decimal place).

Which one of the following statements is correct?

(A) Assertion (1) is TRUE and Assertion (2) is FALSE

(B) Both Assertion (1) and Assertion (2) are TRUE

(C) Both Assertion (1) and Assertion (2) are FALSE

(D) Assertion (1) is FALSE and Assertion (2) is TRUE

Key: (D)

50. The ordinates, u of a 2-hour unit hydrograph (i.e. for 1 cm of effective rain), for a catchment are shown in the table.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Ordinates	0	2	8	18	32	45	30	19	12	7	3	1	0

A 6-hour storm occurs over the catchment such that the effective rainfall intensity is 1 cm/hour for the first two hours, zero for the next two hours, and 0.5 cm/hour for the last two hours. If the base flow is constant at 5 m3/s, the peak flow due to this storm (in m3/s, round off to 1 decimal place) will be ________.

Key: (97)

Sol: For the first two hours $\Rightarrow$ rainfall intensity = 1 cm/hr $\Rightarrow$ rainfall = 2cm

2-4 hours $\Rightarrow 0$

Time	2-hr UH	2hr × Prceci	2hr UH lagged by 2 hr	Lagged by 4 hr R = 2 × 0.5 = 1cm	Total DRH
0	0	0	–	–	0
1	2	4	–	–	4
2	8	18	0	–	16
3	18	36	0	–	36
4	32	64	0	0	64
5	45	90	0	2	92 = Peak
6	30	60	0	8	68
7	19	38	0	32	70
8	12	24	0	45	69
9	7	14	0	30	44
10	3	6	0	19	25
11	1	2	0	12	14
12	0	0	0	7	3

Peak of flood=92+base flow=92+5=97

51. The speed-density relationship of a highway is given as u = 100 – 0.5 k

where, u = speed in km per hour, k = density in vehicles per km. The maximum flow (in vehicles per hour, round off to the nearest integer) is ______.

Key: (5000)

Sol: **Method-I**

Given speed density relationship as $u = 100 - 0.5K$

Capacity (q) $= K.\ u = K(100 - 0.5K) = 100K - 0.5K^2$

For capacity to be maximum, $\frac{dq}{dk} = 0$

$q = 100K - 0.5K^2$

$\frac{dq}{dk} = 100 - 0.5(2K) = 0 \Rightarrow K = 100 veh/km$

Maximum capacity $= 100k - 0.5k^2 = 100 \times 100 - 0.5 \times 100^2$

$= 5000 veh / hr$

Method-2:

Given u=100-0.5k

At the speeds of vehicle zero then, the density will be jam density

$u = 0 \Rightarrow K = K_J$

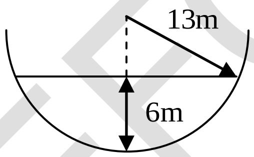
$100 - 0.5K = 0 \Rightarrow K = \frac{100}{0.5} = 200 \text{ veh/hr}$

Similarly $V_f = 100 - 0.5 \times 0 = 100 km/hr$

Maximum capacity$(q_{max}) = \frac{V_f \times K_J}{4} = \frac{100 \times 200}{4}$

$= 5000 veh/hr$

52. Consider the hemi-spherical tank of radius 13 m as shown in the figure (not drawn to scale). What is the volume of water (in m^3) when the depth of water at the centre of the tank is 6 m?



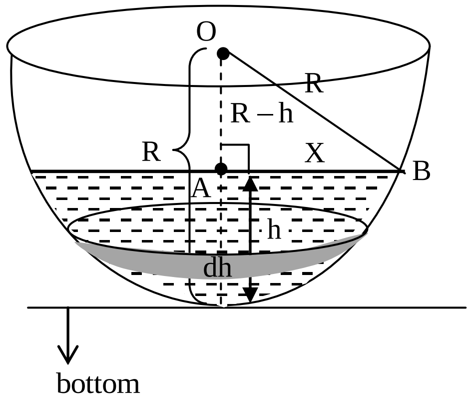
(A) 78π (B) 468π (C) 156π (D) 396π

Key: (D)

Sol: We have

$dv = \pi x^2\ dh$, where $h \in [0, h]$

$\Rightarrow dv = \pi[2Rh - h^2]$, $\left(\begin{array}{l} \because \Delta^{le}OAB \to \text{right angle triangle.} \\ \therefore AB^2 = 0B^2 - 0A^2 \\ \Rightarrow X^2 = R^2 - (R-h)^2 \\ \Rightarrow X^2 = 2Rh - h^2 \end{array}\right)$

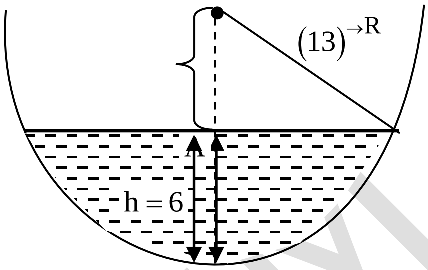


$$\Rightarrow V = \int_0^h \pi\left[2Rh - h^2\right]dh$$

$$= \pi\int_0^h \left[2Rh - h^2\right]dh$$

$$= \pi\left[2R\left(\frac{h^2}{2}\right) - \frac{h^3}{3}\right]_0^h$$

$$\Rightarrow V = \pi\left[Rh^2 - \frac{h^3}{3}\right] \rightarrow (1),$$



where $R \rightarrow$ radius of hemi-sphere

$h \rightarrow$ height from bottom to water level.

$\therefore$ Using (1); we have

$$\text{Required volume, } V = \pi\left[13\left[6^2\right] - \frac{6^3}{3}\right]$$

$$\Rightarrow V = \pi\left[468 - 72\right] = 396\pi.$$

53. A timber pile of length 8m and diameter of 0.2m is driven with a 20 kN drop hammer, falling freely from a height of 1.5m. The total penetration of the pile in the last 5 blows is 40mm. Use the engineering news record expression. Assume a factor of safety of 6 and empirical factor (allowing reducing in the theoretical set, due to energy losses) of 2.5 cm. The safe load carrying capacity of the pile (in kN, round off to 2 decimal places) is ______.

Key: (151.5)

Sol: Uning Engineering News Formula

$$\text{Safe load carrying capacity}\left(Q_s\right) = \frac{Wh}{F(S+C)}$$

Where $W = \text{weight of hammer} = 20\text{kN}$

$h = \text{Height of fall} = 1.5\text{m}$

$S = \text{set value} = \frac{40}{5}\text{mm} = 8\text{mm} = 0.008\text{m}$

$C = 2.5\text{cm} = 0.025\text{m}$

$F = \text{factor of safety} = \text{minimum of 6 to be taken}$

$$Q_{safe} = \frac{Wh}{F(S+C)} = \frac{20\times1.5}{6\left(0.008+0.025\right)} = 151.51\text{kN}$$

54. A 2 m × 4 m rectangular footing has to carry a uniformly distributed load of 120 kPa. As per the 2:1 dispersion method of stress distribution, the increment in vertical stress (in kPa) at a depth of 2 m below the footing is _______.

Key: (40)

Sol: Stress increment $= \dfrac{Q}{(B+Z)(L+Z)}$

$$= \frac{120\times2\times4}{(2+2)(4+2)} = \frac{960}{24} = 40\text{kPa}$$

55. An ordinary differential equation is given below:

$$\left(\frac{dy}{dx}\right)(x\ell nx) = y$$

The solution for the above equation is (Note: K denotes a constant in the options)

(A) $y = K\ell nx$ (B) $y = Kx\ell nx$ (C) $y = Kxe^{x}$ (D) $y = Kxe^{-x}$

Key: (A)

Sol: Given D.E

$$\left(\frac{dy}{dx}\right)(x\ \ell n\ x) = y$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x\ell nn} \to \text{variable} - \text{spearable D.E}$$

Integrating on both sides; then

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1/x}{\ell nx}dx$$

$$\Rightarrow \ell ny = \ell n[\ell nx] + \ell nk; \left[\text{since} \int \frac{f^1(x)}{f(x)}dx = \ell nf(x)\right]$$

$$\Rightarrow \ell ny - \ell nK = \ell n[\ell nx]$$

$$\Rightarrow \ell n[y/k] = \ell n[\ell nx] \Rightarrow \frac{y}{K} = \ell nx$$

$$\Rightarrow y = K\ \ell nx$$